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REMARKS/ARGUMENTS

In response to paragraph 2 of the Office Action (page 2), applicants have now removed all the new matters (equations, functions, matrices, etc) as the Examiner required. Applicants submit that the new Specification includes no new matter, and respectfully request withdrawal of the objection.

In response to paragraph 3 of the Office Action (page 3), a marked up version of the amended application is attached with this submission. Applicants believe this complies with the Examiner's requirement.

In response to paragraph 4 of the Office Action (page 3), applicants have added Figure 1 and Figure 2, so as to make the specification easier to understand. Please note that

- (1) Figure 1 and Figure 2 only show well-known concepts in automatic control; and
- (2) Although the present invention can be clearly described in words as:

"This invention chooses the best values for the tuning parameters in a PID controller or a linear controller in such a way that the largest absolute value of all poles of said discrete-time closed-loop transfer function from said set-point $r(k)$ to said process variable $y(k)$ is minimized subject to, if any, user-specified constraints on one or more of the tuning parameters." (see the Specification or the Claims)

this invention cannot be described by drawings, because it contains abstract mathematical concepts (see the above underlined words) that cannot be described by drawings. However, anyone skilled in the art can easily understand the invention as described in the Specification and Claims and can easily carry out this invention by using well-developed optimization algorithms in mathematics or well-developed computer programs such as the "Optimization Toolbox" in Matlab developed by The MathWorks Inc. (Matlab is a registered trademark of The MathWorks Inc.). The applicants do not need

to teach these well-known prior arts.

In response to paragraph 5 of the Office Action (page 4), applicants submit that the new specification is now in full compliance with the layout suggested by the Examiner.

In response to paragraph 6 of the Office Action (page 8), applicants submit that the new dependant claims now refer to other claims in the alternative only and comply with 37 CFR 1.75 (c). Applicants respectfully request re-examination of the new claims and withdraw of the objections.

In response to paragraphs 7, 8 and 9 of the Office Action (pages 8-9), applicants have removed the equations, transfer functions, and matrices as required by the Examiner. Anyone skilled in the art knows how to find the closed-loop or open-loop transfer functions, the poles of a transfer function, the definition of PID controllers and linear controllers and their structures, and the use of well-developed optimization algorithms in mathematics and optimization computer programs such as the "Optimization Toolbox" in Matlab developed by The MathWorks Inc. to solve the optimization problem formulated in the present invention (Matlab is a registered trademark of The MathWorks Inc.). The applicants do not need to teach these well-known prior arts. For example, if an invention needs the calculation of the value of $\sin(x)$, the inventor does not need to teach the definition of $\sin(x)$, neither does he need to teach the various mathematical methods, algorithms or computer programs for finding the value of $\sin(x)$, since these are all well-known prior arts. In Matlab, the function "fminimax" can be used to directly solve the optimization problem (minimax problem) formulated in this invention (see the newly cited book "Optimization Toolbox User's Guide" authored by Coleman et al. that comes with the commercial software product "Optimization Toolbox" developed by The MathWorks Inc). Matlab and the associated "Optimization Toolbox" are well known among people skilled in the art. Therefore, applicants do not need to teach how to minimize the maximum of absolute values of all poles of a discrete time closed-loop transfer function from a set point to a process variable.

Therefore, applicants respectfully submit that the new claims have overcome the defects as pointed out by the Examiner and are in full compliance with 35 U.S.C. 112 since anyone skilled in the art can

implement this invention without any difficulty. Applicants respectfully request re-examination of the new claims and withdrawal of the rejections and criticisms as stated in paragraphs 7, 8 and 9 of the Office Action.

In response to paragraph 10 of the Office Action (page 9), applicants revised the specification so that it now relies on the prior art of record cited.

Applicants have read through the following new reference cited by the Examiner:

A. Cichocki et al., "Neural Networks for Solving Systems of Linear Equations II: Minimax and Least Absolute Value Problems", IEEE Transactions on Circuits and Systems II: Analog and Digital Signal Processing, Volume 39, Issue 9, Sept. 1992, Pages 619-633.

A careful study of this prior art shows it deals with the problem of solving a system of linear equations by minimizing the largest absolute error using the neural networks technology. It is not apparent that this method can be used to solve the minimax problem formulated in this invention since the minimax problem in this invention is totally different. However, there are many well-developed optimization methods in mathematics that can directly solve the minimax problem formulated in this invention (see, e.g., the many cited references mentioned in the "DETAILED DESCRIPTION OF THE INVENTION" section). Indeed many computer software packages such as the "Optimization Toolbox" in Matlab developed by The MathWorks Inc. can solve the minimax problem formulated in this invention without any difficulty. Matlab and its various toolboxes such as the "Optimization Toolbox" are very popular among people skilled in the art. All these well-developed optimization methods are well-known prior art and do not need to be taught in this invention.

Applicants respectfully submit that the new specification now relies on the cited references and therefore complies with the requirements in paragraph 10 of the Office Action.

Conclusion

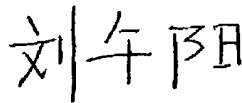
In order to comply with the Office Actions, Applicants have re-written the specification and claims. For all the aforesaid reasons applicants submit that the new specification and claims do not contain new matters. Drawings are added. The layout of the specification is now as suggested by the Examiner. The specification and claims enable anyone skilled in the art to carry out the invention without any difficulty. Applicants respectfully submit that this application is now in full condition for allowance.

However, applicants are willing to accept any suggestions that the Examiner believes necessary in order to avoid further proceedings.

Very respectfully,



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VERSION WITH MARKINGS TO SHOW CHANGES MADEIn the Specification:

Please cancel the old specification in record and replace it with the new Specification (underlined) as follows:

DescriptionTITLE OF THE INVENTION

Methods of Designing Optimal PID Controllers

TECHNICAL FIELD OF THE INVENTION

This invention relates to the design of the structure of a multivariable PID controller and the optimal choice of its PID parameters.

BACKGROUND OF THE INVENTION

A traditional PID controller is used to control an industrial process. The process variable (PV) goes into the PID controller, which calculates the controller output (CO) according to a PID control equation. This CO is then converted to an analog signal, which is sent to the process so that the said PV can track a user specified value called set point (SP). The said SP can change with time. The performance of a PID controller depends on the choice of its three PID parameters. For independent form of PID controllers these three PID parameters are the proportional gain K_p , the integral gain K_i , and the derivative gain K_d . For dependent form of PID controllers these three PID parameters are the gain K , integral time T_i , and derivative time T_d . In traditional PID controllers the said PV, SP, CO, and PID parameters are all scalars. We call this kind of PID controllers the single input single output (SISO) PID controllers. The Ziegler-Nichols PID controller tuning method is the major one of the many methods for finding the values of PID parameters.

DETAILED DESCRIPTION OF THE INVENTION

In this invention the SISO PID controller is extended to the multiple-input multiple-output (MIMO) PID controller that has n process variables $PV1, PV2, \dots$, and PVn and m controller outputs $CO1, CO2, \dots$, and COm , where m and n are positive integers. Corresponding to $PV1, PV2, \dots$, and PVn there are n set points $SP1, SP2, \dots$, and SPn . In this case PV becomes a vector with $PV1, PV2, \dots$, and PVn being its first, second, \dots , and n -th component, CO becomes a vector with $CO1, CO2, \dots$, and COm being its first, second, \dots , and m -th component, SP becomes a vector with $SP1, SP2, \dots$, and SPn being its first, second, \dots , and n -th component, and the PID control equation becomes $CO(k) = CO(k-1) + K1 \cdot SP(k) \cdot T + K1 \cdot a(k,1) + K2 \cdot a(k,2) + \dots + Kj \cdot a(k,j)$, where k is the discrete time, T is the sampling period, j is a positive integer, $K1, K2, \dots, Kj$ are m by n PID parameters, $a(k,1) = [PV(k)] \cdot T$, and $a(k,j) = [a(k,j-1) - a(k-1,j-1)]/T$ for $j > 1$. It is important to note that—

1. an MIMO PID controller is able to take into account the interaction among the n process variables and m controller outputs, which can not be achieved by simply applying SISO PID controllers to each of the n process variables, and—
2. there is no set point in any of $a(k,1), a(k,2), \dots$, and $a(k,j)$, which can avoid the unwanted sudden change in CO when SP changes with time.—

The next problem of designing the optimal PID controller is to find the best values for the PID parameters $K1, K2, \dots$, and Kj . An optimization based method for solving this problem consists of the following four steps:—

1. Convert the PID control equation into discrete time form if it is not in discrete time form.—
2. Build a discrete time linear model for the process that is to be controlled by the said PID controller.—
3. Form the discrete time closed loop transfer function from said vector SP to said vector PV .—
4. Find the best PID parameters by using an optimization algorithm which minimizes the largest modulus of all poles of the discrete time closed loop transfer function obtained at step 3, where the modulus of a pole is defined to be the absolute value of the complex number which represents the pole. If the PID parameters are subject to some constraints, then a constrained optimization algorithm can be used which minimizes the largest modulus of all poles of the discrete time closed loop transfer function

~~obtained at step 3 and at the same time guarantees that all user prescribed constraints on the PID parameters are satisfied.~~

~~PID controllers with their parameters so obtained guarantee that PV can track SP quickly~~

SPECIFICATION

TITLE OF THE INVENTION

Methods of Designing Optimal Linear Controllers

CROSS-REFERENCE TO RELATED APPLICATIONS

Domestic priority data: This application is a 371 of PCT/IB01/01002 06/07/2001

Foreign applications: June 20, 2000 [CA] Canada 2,311,268

STATEMENT REGARDING FEDERALLY SPONSORED RESEARCH OR DEVELOPMENT

Not Applicable.

INCORPORATION-BY-REFERENCE OF MATERIAL SUBMITTED ON A COMPACT DISC

Not Applicable

BACKGROUND OF THE INVENTION

Technical Field of the Invention

This invention relates to the design of optimal linear controllers and PID controllers.

Description of Related Art

Figure 1 shows a process 1 controlled by a PID (proportional-integral-derivative) or a linear controller 2. The PID controller or linear controller 2 means a linear system, usually implemented in a computer or as an electronic circuit, that receives the process variable signal $y(k)$ and set-point signal $r(k)$ as its inputs and calculates the controller output signal $u(k)$ according to a PID control equation or a linear control equation, where k is the integer discrete time variable. For a multiple-input and

multiple-output (MIMO) process, the process variable $y(k)$ is an n -dimensional vector with each of its components being a scalar process variable and the controller output $u(k)$ is an m -dimensional vector variable with each of its components being a scalar controller output, where n and m are positive integers. It is desired that the performance of the controller should be such that, after the controller output signal $u(k)$ is sent to the process 1, the process variable $y(k)$ should approach the set-point $r(k)$ quickly and smoothly as the time variable k increases, where $r(k)$ is an n -dimensional set-point signal (also known as the reference signal or the command signal, etc.)

There are many types of PID controllers, depending on the use of different types of PID control equations. All types of PID controllers can be viewed as special cases of linear controllers. By definition, a discrete-time linear controller means a controller whose output $u(k)$ is a linear function of past controller outputs, current and past process variables, and current and past set-point variables, as shown in Figure 2. This definition is well known to any person skilled in the art (see, e.g., Figure 2.4 and Assumption 2.3 on page 29 of the cited book "Linear Controller Design" authored by Boyd and Barratt and published by Prentice Hall in 1991).

Once the structure of its control equation is properly selected, the performance of a PID controller or a linear controller depends mainly on the choice of coefficients in the control equation. The adjustable numbers in the coefficients are called tuning parameters (also known as tuning coefficients, tuning gains or tuning constants, etc.)

How to properly choose tuning parameters for a PID controller is a problem that has attracted a lot of studies ever since PID controllers became widely used in industry in the early 1940s. The Ziegler-Nichols tuning methods developed by Ziegler and Nichols in 1942 (see the Ziegler and Nichols reference), either in their original form or in some modification, are still widely used in industry. Other model-based optimization methods choose the tuning parameters by minimizing some well-known control performance index such as the integrated absolute errors (IAE), the integrated squared errors (ISE), the integrated time absolute error (ITAE), etc. (see US patent 5,453,925). However, practice shows that all these methods often lead to the undesired oscillatory control results.

Many tuning methods are based on the continuous-time transfer function analysis (see, for example, the cited US patents 6,434,436 B1 to Adamy et al., 5,866,861 to Rajamani et al., 4,539,633 to Shigemasa et al., 4,563,734 to Mori et al., the cited research papers by Celentano et al., Zhuang et al.,

Dorf et al., Katebi et al., and Saeki et al., and the cited book "Linear Controller Design" authored by Boyd et al.). Therefore these methods cannot guarantee the optimal performance of controllers in the discrete-time domain. This is a real problem since nowadays almost all PID or linear controllers are implemented in digital computers and therefore they are working in the discrete-time domain, not the continuous-time domain.

Other tuning methods in discrete-time domain cannot guarantee that the largest absolute value of all the poles of the closed-loop transfer function is minimized. Therefore they cannot guarantee the optimal performance of the controller (see, e.g., the cited US patent No. 5,680,304 to Wang et al. and the cited research paper by Yamamoto et al., etc.).

BRIEF SUMMARY OF THE INVENTION

This invention chooses the tuning parameters in a PID controller or a linear controller by minimizing the maximum of absolute values of all poles of the discrete-time closed-loop transfer function from said set-point $r(k)$ to said process variable $y(k)$ subject to, if any, user-specified constraints on one or more of the tuning parameters. When the tuning parameters are chosen this way, the PID controller or linear controller can guarantee that the process variable $y(k)$ tracks the set-point $r(k)$ smoothly and quickly as time k increases.

BRIEF DESCRIPTION OF THE DRAWINGS

Figure 1 shows a process 1 controlled by a PID controller or a linear controller 2. The PID controller or linear controller is in discrete-time form.

Figure 2 shows a PID or a linear controller. By definition, the controller output $u(k)$ of a linear controller is a linear function of $u(k-1)$, $u(k-2)$, ..., $u(k-a)$, $y(k)$, $y(k-1)$, ..., $y(k-b)$, $r(k)$, $r(k-1)$, $r(k-2)$, ..., $r(k-c)$, wherein $a > 0$, b and c are integers (This definition is well known to any person skilled in the art and is given in Figure 2.4 and Assumption 2.3 on page 29 of the cited book "Linear Controller Design" authored by Boyd et al.)

DETAILED DESCRIPTION OF THE INVENTION

From now on it is always assumed that:

- (1) The open-loop discrete-time transfer function of the process 1 is known, and
- (2) The structure of the control equation of the PID controller or the linear controller has already been properly chosen and is already in discrete-time form.

It is then easy for anyone skilled in the art to find the discrete-time closed-loop transfer function from the set-point $r(k)$ to the process variable $y(k)$, as shown in Figure 1. This invention chooses the best values for the tuning parameters in a PID controller or a linear controller in such a way that the largest absolute value of all poles of said discrete-time closed-loop transfer function from said set-point $r(k)$ to said process variable $y(k)$ is minimized subject to, if any, user-specified constraints on one or more of the tuning parameters. This choice guarantees that the process variable $y(k)$ tracks the set-point $r(k)$ smoothly and quickly as time k increases.

The above description contains a minimax optimization problem. The general minimax problem has been very well studied and many successful numerical methods and algorithms have been developed, see, e.g., the cited publications by Charalambous et al., Conn et al., Conn, Pillo, Gigola et al., Polyak, Polak, Polak et al., Vardi, Zang, Murray et al., Kaufman et al., Reemsten, Zhou et al., and Laskari et al. Successful commercial computer programs such as the "Optimization Toolbox for use with Matlab" developed by The MathWorks Inc. can directly be used to solve the minimax problem as formulated in this invention without any difficulty (see the cited book "Optimization Toolbox User's Guide" authored by Coleman et al. and published by The MathWorks Inc.). The "Optimization Toolbox" and Matlab have been well known among people skilled in the art. It is easy for anyone skilled in the art to solve the minimax problem directly using the "Optimization Toolbox", the methods in the cited publications mentioned above, or any other method.

In the Claims:

Please cancel all claims in record and replace them with the new claims 22-33 (underlined) as follows:

We claim:

1. A MIMO (multiple input multiple output) PID controller which has an n-dimensional process variable vector PV with the n process variables PV_1, PV_2, \dots , and PV_n being its first, second, \dots , and n-th

~~component, an n-dimensional set point vector SP with the n set points SP1, SP2, . . . , and SPn being its first, second, . . . , and n-th component, and an m-dimensional controller output vector CO with the m controller outputs CO1, CO2, . . . , and COM being its first, second, . . . , and m-th component, where m and n are positive integers, and in which the PID control equation is $CO(k) = CO(k-1) + K1 \cdot SP(k) \cdot T + K1 \cdot a(k,1) + K2 \cdot a(k,2) + \dots + Kj \cdot a(k,j)$, where k is the discrete time, T is the sampling period, j is a positive integer, K1, K2, . . . , Kj are m by n PID parameters, $a(k,1) = [-PV(k)] \cdot T$, and $a(k,j) = [a(k,j-1) - a(k-1,j-1)]/T$ for $j > 1$.~~

~~2. An MIMO PID controller of claim 1, in which the m by n PID parameters K1, K2, . . . , and Kj are obtained by using an optimization algorithm which minimizes the largest modulus of all poles of the discrete time closed loop transfer function from said SP to said PV.~~

~~3. An MIMO PID controller of claim 2, wherein the said optimization algorithm is a constrained optimization algorithm which minimizes the largest modulus of all poles of the discrete time closed loop transfer function from said SP to said PV and at the same time guarantees that the user prescribed constraints on the PID parameters are satisfied.~~

~~4. An MIMO PID controller of claim 1, wherein some or all of the terms $K2 \cdot a(k,2)$, $K3 \cdot a(k,3)$, . . . , and $Kj \cdot a(k,j)$ that appear on the right hand side of the PID control equation are removed, for example, a PID controller with its control equation being $CO(k) = CO(k-1) + K1 \cdot SP(k) \cdot T + K1 \cdot a(k,1) = CO(k-1) + K1 \cdot [SP(k) - PV(k)] \cdot T$, which is also called a 1-only controller, and a PID controller with its control equation being $CO(k) = CO(k-1) + K1 \cdot SP(k) \cdot T + K1 \cdot a(k,1) + K2 \cdot a(k,2) = CO(k-1) + K1 \cdot [SP(k) - PV(k)] \cdot T - K2 \cdot [PV(k) - PV(k-1)]$, which is also called a PI controller, etc.~~

~~5. An MIMO PID controller of claim 4, wherein the remaining PID parameters are obtained by using an optimization algorithm which minimizes the largest modulus of all poles of the discrete time closed loop transfer function from said SP to said PV.~~

~~6. An MIMO PID controller of claim 5, wherein the said optimization algorithm is a constrained~~

~~optimization algorithm which minimizes the largest modulus of all poles of the discrete time closed loop transfer function from said SP to said PV and at the same time guarantees that the user prescribed constraints on the PID parameters are satisfied.~~

~~7. A PID controller of claim 1, wherein said PV, said SP, said CO, and said PID parameters are all scalars, and $m=n=1$.~~

~~8. A PID controller of claim 2, wherein said PV, said SP, said CO, and said PID parameters are all scalars, and $m=n=1$.~~

~~9. A PID controller of claim 3, wherein said PV, said SP, said CO, and said PID parameters are all scalars, and $m=n=1$.~~

~~10. A PID controller of claim 4, wherein said PV, said SP, said CO, and said PID parameters are all scalars, and $m=n=1$.~~

~~11. A PID controller of claim 5, wherein said PV, said SP, said CO, and said PID parameters are all scalars, and $m=n=1$.~~

~~12. A PID controller of claim 6, wherein said PV, said SP, said CO, and said PID parameters are all scalars, and $m=n=1$.~~

~~13. A method of finding the optimal PID parameters or tuning constants for any PID controller or a linear controller by minimizing the largest modulus of all poles of the discrete time closed loop transfer function from set point SP to process variable PV.~~

~~14. A method of Claim 13, wherein the optimal PID parameters or tuning constants are obtained by minimizing the largest modulus of the discrete time closed loop transfer function from said SP to said PV subject to the constraint that some or all of the PID parameters or tuning constants are within user~~

~~specified admissible ranges.~~

~~15. A PID controller of Claim 1 or any PID controller or linear controller, wherein the PID parameters or tuning constants are obtained such that~~

- ~~(1) the maximum real part of all poles of the continuous-time closed-loop transfer function from said SP to said PV is minimized, possibly subject to any user specified constraints on the PID parameters or tuning constants, such as some or all of elements in the PID parameters or tuning constants should be within user specified ranges, where the said PID parameters or tuning constants can be matrices or scalars, or~~
- ~~(2) the maximum magnitude of some or all elements in the PID parameters or tuning constants is minimized, subject to the constraint that the maximum real part of all poles of the said continuous-time closed-loop transfer function from said SP to said PV is not larger than a user specified number.~~

We claim:

22. A method for determining the optimal tuning parameters in a linear controller, wherein

- 1) said controller receives an n-dimensional process variable signal $y(k)$ from a process and an n-dimensional set-point signal $r(k)$, calculates an m-dimensional controller output $u(k)$ according to a linear control equation, and sends said $u(k)$ to said process, where k is the integer discrete time variable and n and m are positive integers;
- 2) said tuning parameters are the adjustable numbers in the coefficients in said linear control equation that are to be determined; and
- 3) said method finds the optimal values for said tuning parameters by minimizing the maximum of absolute values of all poles of the discrete-time closed-loop transfer function from said set-point $r(k)$ to said process variable $y(k)$.

23. A method as in Claim 22, wherein said minimization of the maximum of absolute values of all poles of said discrete-time closed-loop transfer function is subject to user-specified constraints placed on one or more of said tuning parameters.

24. A method as in Claim 22, wherein said controller output $u(k) = u(k-1) + K_1 * r(k) * T + K_1 * a(k,1) +$

- $K_2 * a(k,2) + \dots + K_p * a(k,p)$, wherein k is the discrete time variable, $*$ is the multiplication operator, T is the sampling period, p is a positive integer, the m by n matrices K_1, K_2, \dots , and K_p are tuning parameters, $a(k,1) = [-y(k)] * T$, and $a(k, p) = [a(k,p-1) - a(k-1,p-1)] / T$ for $p > \text{or } = 2$.
25. A method as in Claim 23, wherein said controller output $u(k) = u(k-1) + K_1 * r(k) * T + K_1 * a(k,1) + K_2 * a(k,2) + \dots + K_p * a(k,p)$, wherein k is the discrete time variable, $*$ is the multiplication operator, T is the sampling period, p is a positive integer, the m by n matrices K_1, K_2, \dots , and K_p are tuning parameters, $a(k,1) = [-y(k)] * T$, and $a(k, p) = [a(k,p-1) - a(k-1,p-1)] / T$ for $p > \text{or } = 2$.
26. A method as in Claim 22, wherein said linear controller is a PID (proportional-integral- derivative) controller.
27. A method as in Claim 23, wherein said linear controller is a PID controller.
28. A linear controller as in Claim 22 with its tuning parameters determined therein.
29. A linear controller as in Claim 23 with its tuning parameters determined therein.
30. A linear controller as in Claim 24 with its tuning parameters determined therein.
31. A linear controller as in Claim 25 with its tuning parameters determined therein.
32. A PID controller as in Claim 26 with its tuning parameters determined therein.
33. A PID controller as in Claim 27 with its tuning parameters determined therein.

In the Abstract

Please cancel the old Abstract in record and replace it with the new Abstract (underlined) as follows:

~~Methods of designing the structure of multiple-input multiple-output (MIMO) PID controllers and methods of finding the optimal values for the MIMO PID parameters are disclosed. The optimal values of MIMO PID parameters are obtained by using an optimization algorithm which minimizes the largest modulus of all poles or the discrete time closed-loop transfer function from set point SP to process variable PV, with or without user prescribed constraints on the PID parameters. Methods of designing the structure of single-input single-output (SISO) PID controllers and methods of finding tile optimal values for SISO PID parameters are also disclosed as special MIMO PID controller cases.~~

Methods of designing optimal discrete-time PID (proportional-integral-derivative) controllers and linear

controllers are disclosed. The optimal values of the tuning parameters in a PID controller or a linear controller are determined by minimizing the maximum of absolute values of all poles of the discrete-time closed-loop transfer function from the set-point to the process variable subject to, if any, user-specified constraints on one or more of the tuning parameters.

In the drawings:

Please add the attached Figure 1 and Figure 2.